Advanced Mathematical Models & Applications
Vol.9, No.1, 2024, pp.121-129
https://doi.org/10.62476/amma9121

PARTITION DIMENSION FOR LINE GRAPH OF HONEYCOMB NETWORKS AND AZTEC DIAMOND NETWORKS

R. Nithya Raj[1,*], **R. Sundara Rajan**[1], **Thatchinamoorthi Ragunathan**[2], **N. Muthuvairavan Pillai**[3], **S. Balamuralitharan**[4]

[1]Department of Mathematics, Hindustan Institute of Technology and Science, Chennai 603 103, India
[2]Department of Basic Sciences, SRM College of Agricultural Sciences, SRM Institute of Science and Technology, Chennai 603201, India
[3]Computer Science and Business Systems, R.M.D Engineering College, Kavaraipettai - 601206, Thiruvallur, Tamil Nadu, India
[4]New Research and Innovation Society (new rains), Chennai, Tamil Nadu, India

Abstract. In graph theory, a commonly used concept is the partition dimension or partition metric basis is to uniquely identify the node set of a structure by dividing it into smaller subsets, known as partition resolving sets. These subsets can then be used to define the partition dimension or partition metric of the graph. This concept is useful in the analysis and understanding of the structure and properties of graphs. This article describes a partition dimension of the line graph of the honeycomb network, the Aztec diamond network, and the extended Aztec diamond network.

Keywords: Partition dimension, honeycomb networks, Aztec diamond network, extended Aztec diamond networks.

AMS Subject Classification: 05C12, 05C25.

***Corresponding author:** R. Nithya Raj, Department of Mathematics, Hindustan Institute of Technology and Science, Chennai 603 103, India, e-mail: *nithyarajmaths@gmail.com*

Received: 12 October 2023; Revised: 21 November 2023; Accepted: 12 December 2023;
Published: 22 April 2024.

1 Introduction

Mathematical chemistry has recently introduced a wide range of applications to the understanding of the chemical structures that underlie the existing chemical idea using mathematical principles and chemistry techniques, as well as developing and exploring new mathematical models of chemical phenomena. In chemistry, a chemical graph's nuclear assistant properties can be expressed using chemical graph theory. It is expected that chemical graph theory will be important in the description and organization of any frame or component structure created. Chemical compounds or graphs are graphical representations consisting of nodes, which are atoms, and lines, which are atom-to-atom bonds.

The terms "resolvability" and "placement" in a graph are explained by Harary & Melter (1976). In another work, Chartrand et al. (1998) obtained a connection between the graph's

How to cite (APA): Nithya Raj, R., Sundara Rajan, R., Ragunathan, T., Muthuvairavan Pillai, N. & S. Balamuralitharan (2024). Partition dimension for line graph of honeycomb networks and aztec diamond networks. *Advanced Mathematical Models & Applications*, *9*(1), 121-129 https://doi.org/10.62476/amma9121

metric and partition dimension. There are numerous fields where partition dimension is used, including a network's validation (Beerliova et al., 2006), navigating by robots (Khuller et al., 1996), techniques for mastermind games Chvatal (1983), and for describing chemical compounds in chemistry (Chartrand et al., 2000), or to issues with image processing and pattern identification, some of which need the use of hierarchical data structures (Melter & Tomescu, 1984). Also see Harary & Melter (1976); Chartrand et al. (2000); Rajan et al. (2012) for applications in networks.

The metric dimension of honeycomb networks is 3, as demonstrated by Manuel et al. (2008). Graphs of honeycomb networks' partition dimensions were explored in Rajan et al. (2012). In Monica & Santhakumar (2019) studied the partition dimension of rooted product graphs. Nithya & Elavarasi (2022) investigated partition and local metric dimensions for an extended annihilating-ideal graph. In Hussian & Farooq (2019) obtained the metric dimension for the line graph of honeycomb networks. Motivated by the above literature survey of honeycomb networks we are analyzing the honeycomb network's line graph's partition dimension.

2 Basis Concepts

We consider connected, simple, and undirected graph A having node set $V(A)$ and lines set $E(A)$. The length of the shortest path, among two nodes $l, m \in V(A)$ is represented by the symbol $d(l, m)$ and refers to the number of edges that connect them in the shortest distance. The distance between a node u and a subset Q of $V(A)$ is defined as $d(u, Q) = min\{d(u, q); q \in Q\}$. Let $\pi = \{P_1, P_2, P_3, \ldots, P_k\}$ being organised partition of $V(A)$. Now partition an illustration of a vertex $x \in V(A)$ with respect to π is $r(a/(\pi) = (d(a, P_1), d(a, P_2), \ldots, d(a, P_k))$. If for each set of two unique vertices $a, b \in V(A)$, we have $r(a/\pi) \neq r(b/\pi)$, π is a partition that resolves is known as a resolving partition, and its least cardinality is known as a resolving partition of $V(A)$, and it is symbolized by the symbol $pd(A)$.

In studies about the partition dimension of graphs, established by many authors (Koam et al., 2022; Hasmawati et al., 2022; Azeem et al., 2022). The limits on the partition dimension of trees and unicyclic are discussed in Velazquez et al. (2014,?). The wheel graph's partition dimension was discovered by Tomescu et al. (2007). The partition dimension for honeycomb, hexagonal, and rooted product graphs in Rajan et al. (2012); Monica & Santhakumar (2019).

We need the following definitions to prove our main results.

Definition 1. *The line graph of a given graph, abbreviated as $L(A)$, is defined as follows. The edges of G are represented by the vertices of $L(A)$. In A, two edges are terminated at the same vertex if and only if the corresponding vertices are adjacent in $L(A)$. Figure 1 provides an example of a network A and its related line network $L(A)$.*

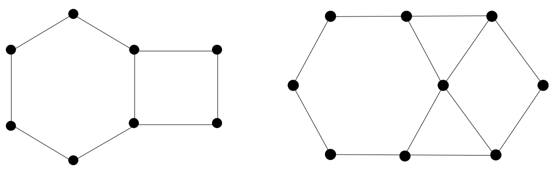

Figure 1: A network A and its line network $L(A)$

Definition 2. *The graph's metric dimension is the minimum number of vertices required to label the graph in such a way that the label of any two vertices uniquely determines their distance from each other, and it is denoted by $dim(A)$.*

A representation of the relationship between $dim(A)$ and $pd(A)$ for every non-trivial connected graph A can be found in Chartrand et al. (1998), $pd(A) \leq dim(A) + 1$.

Finding a graph's partition dimension is made much easier by the theorems that follow.

Theorem 1. *Chartrand et al. (1998) Let $x_1, x_2 \in V(A)$, and π be a resolving partition of $V(A)$, if all the nodes $y \in V(A) \backslash (x_1, x_2)$ then x_1 and x_2 belongs to varies classes of π.*

Theorem 2. *Chartrand et al. (1998) Assume that A is a connected simple network with no isolated vertices.*

i) If A is a path graph with $n \geq 2$ vertices, then $pd(A) = 2$, and conversely, if $pd(A) = 2$, then A is a path graph with $n \geq 2$ vertices.

ii) If A is a complete graph, then $pd(A) = n$, and conversely, if $pd(A) = n$, then A is a complete graph K_n.

In computing the partition dimension of connected graphs, the aforementioned results are helpful. Zhang et al. (2022) calculated the metric dimension of the subdivision of honeycomb networks and Aztec diamond networks. In this study, we calculate the partition dimension of the line graph of honeycomb networks, Aztec diamond networks, and extended Aztec diamond networks.

The partition dimension of a line graph of a honeycomb network will be addressed in Section 3, the partition dimension of an Aztec diamond network and its line graph will be studied in Section 4, and the partition dimension of an extended Aztec diamond network will be discussed in Section 5. The concluding remarks and open problems are listed in Section 6.

3 Partition dimension of the line graph of honeycomb networks

First, the structural introduction to HCN_n and $L(HCN_n)$ is provided in this section. Secondly, we have shown that the partition dimension of $L(HCN_n)$ is 3.

Honeycomb networks are a type of regular network that is often used in the study of complex systems and their properties. These networks are characterized by their hexagonal topology and can be constructed in any dimension. A honeycomb network HCN_n can be constructed in a variety of ways using polygons, where n denotes how many hexagons there are the network has from its center to its edge. The given HCN_1 needs to have a layer of six hexagons added to its outer edge to create HCN_2. Therefore, in order to create HCN_n, we coat HCN_{n-1} with $6(n-1)$ hexagons. To obtain the line graph $L(HCN_n)$ of the honeycomb network HCN_n, we represent each edge in HCN_n as a vertex in $L(HCN_n))$, and two vertices in $L(HCN_n)$ are adjacent if their corresponding edges in HCN_n share an endpoint. The upper bound of radio number and radio mean number for honeycomb and honeycomb torus networks are found in Augustine & Roy (2023). Roshini et al. (2021) studied Non-neighbor topological indices for Honeycomb networks. Honeycomb networks and their line graphs have several interesting properties, including high symmetry, regularity, and low degree of divergence, which make them useful in a variety of applications, including materials science, physics, and computer science. Figure 2 shows a diagram of a honeycomb network and a line graph of the same dimensions.

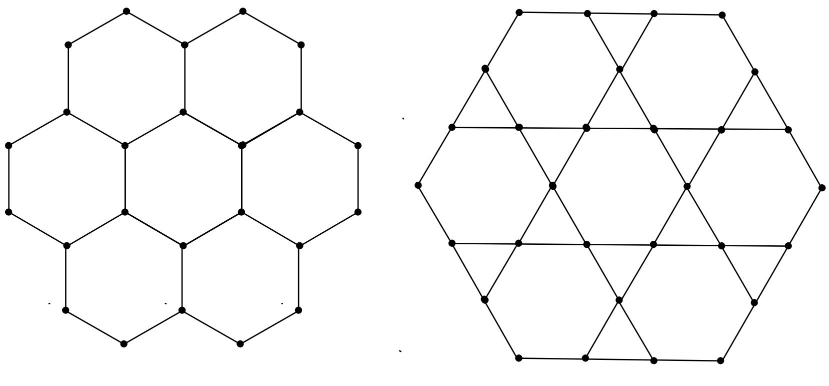

Figure 2: Honeycomb networks and its line graph of dimension 2

Theorem 3. *Let A be a honeycomb network, then $pd(L(A)) = 3$ for $n \geq 2$.*

Proof. The line graph of honeycomb networks has 7 levels, say, a_i^j, b_i^j, c_i^j, d_i^j, l_i, r_i and m_i. Label the vertices of $\{a_i^j, c_i^j : 1 \leq i \leq 4n-2j-2, 1 \leq j \leq n-1\}$, $\{b_i^j, d_i^j : 1 \leq i \leq 2n-2j+1, 1 \leq j \leq n-1\}$ and $\{l_i, r_i : 1 \leq i \leq 4n-2\}$ and $\{m_i : 1 \leq i \leq 2n\}$. The identification of $L(HCN_n)$ is shown in Figure 3.

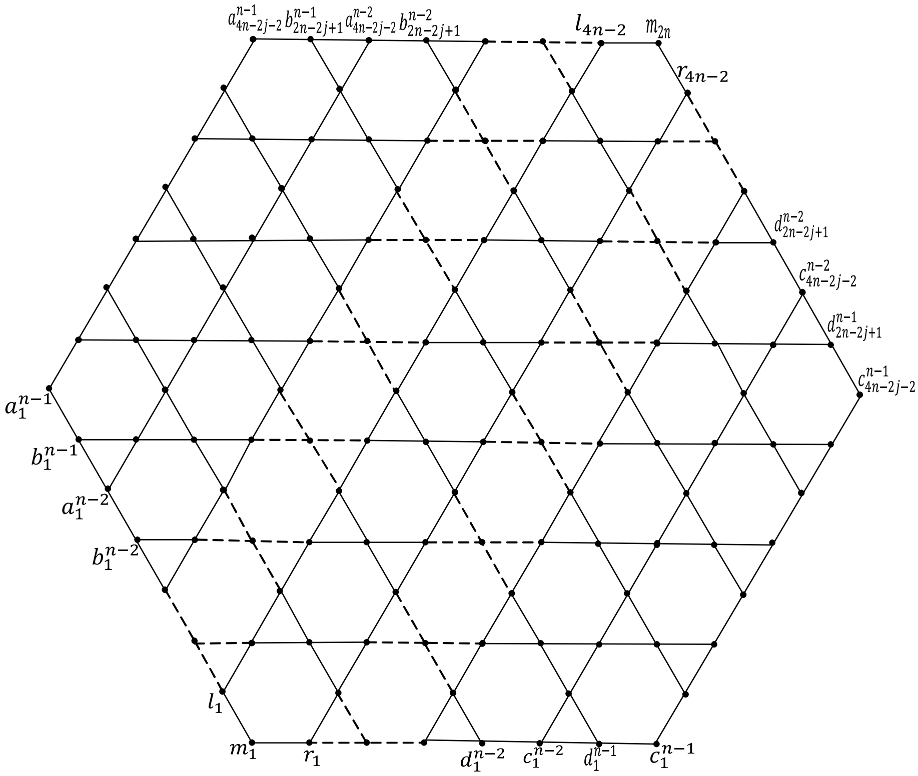

Figure 3: Labeling of the line graph of honeycomb networks of dimension n

By Theorem 4, we have $pd(L((HCN_n))) \geq 3$. We will prove the equality with the following partition-resolving sets. Let $\pi = \{P_1, P_2, P_3\}$ where $P_1 = \{m_1\}$, $P_2 = \{a_1^{n-1}, a_2^{n-1}, \ldots, a_{4n-2j-2}^{n-1}\}$ and $P_3 = V(A) - \{P_1 \cup P_2\}$ be the resolving partition set of $L(A)$. A vertex's representation in $L(A)$ concerning π is as follows.

For $2 \leq i \leq 2n$ each vertex's visual representation m_i of $L(A)$ in relation to π as $r(m_i \backslash \pi) = (2i-1, 2n-1, 0)$.

For $1 \leq i \leq 4n-2$ each vertex's visual representation l_i of $L(A)$ in relation to π as $r(l_i \backslash \pi) = (i, 2n-2, 0)$.

For $1 \leq i \leq 4n-2$ each vertex's visual representation r_i of $L(A)$ in relation to π as $r(r_i \backslash \pi) = (i, 2n-2n, 0)$.

For $1 \leq i \leq 4n-2j-2$, $1 \leq j \leq n-2$ the representation each vertex a_i^j, of $L(A)$ with respect to π as $r(a_i^j \backslash \pi) = (2j+i, 2n-2j-2, 0)$.

For $1 \leq i \leq 4n-2j-2$, $1 \leq j \leq n-1$ the representation each vertex c_i^j of $L(A)$ with respect to π as $r(c_i^j \backslash \pi) = (2j+i, 2n+2j, 0)$.

For $1 \leq i \leq 2n-2j-2$, $1 \leq j \leq n-1$ the representation each vertex b_i^j and d_i^j of $L(A)$ with respect to π as follows.

$r(b_i^j \backslash \pi) = (2(j+i)-2, 2n-2j-1, 0)$ and $r(d_i^j \backslash \pi) = (2(j+i)-2, 2n+2j-1, 0)$.

Each vertex's position concerning the above representation of the π we get $r(u \backslash \pi) \neq r(v \backslash \pi)$ for any $u, v \in L(A)$. Hence the $pd(L(A)) = 3$. $\square$

4 Partition dimension of the Aztec diamond networks

In this section, we've demonstrated that, for $n \geq 2$, the Aztec diamond networks and its line graph partition dimension is 3.

The Aztec diamond graph of order n, denoted by AZN_n, is formed by connecting the straight edges of staircase shapes of height n. Thus, it can be described as a lattice. composed of unit squares centred at (l, m) such that $|l| + |m| \leq n$. The number of unit squares that make up AZN_n with order n is $2n(n+1)$. In Fendler & Grieser (2015); Kokhas (2009), an AZN_n with various proportions is illustrated and further investigated. For illustration in following Figure 4 depicts AZN_2 and AZN_3, respectively.

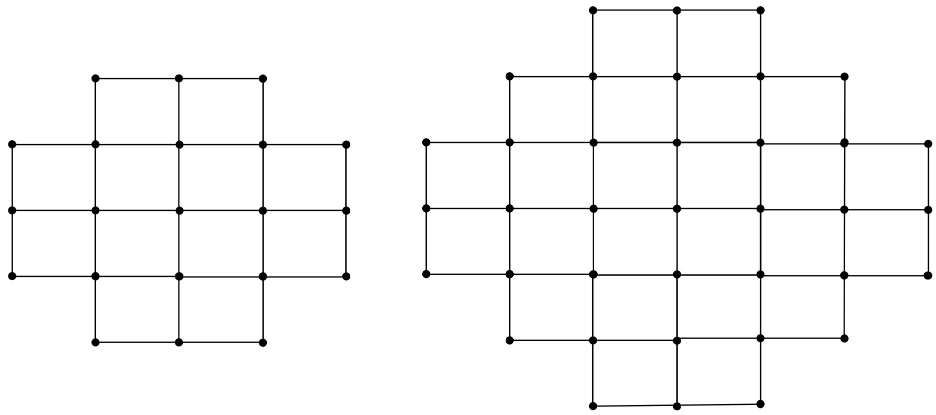

Figure 4: Aztec diamond networks of dimension 2 and 3

Theorem 4. *Let A be a Aztec diamond networks of dimension n, then $pd(A) = 3$, $n \geq 2$.*

Proof. Since the vertex set of AZN_n has 5 levels, say l_i, r_i, p_i^j, q_i^j, and m_i. Label the vertices $\{p_i^j, q_i^j, : 1 \leq i \leq 2n-2j+1,\ 1 \leq j \leq n-1\}$, $\{l_i, r_i : 1 \leq i \leq 2n+1\}$ and $\{m_i : 1 \leq i \leq 2n+1\}$. The identification of AZN_n is shown in Figure 5.

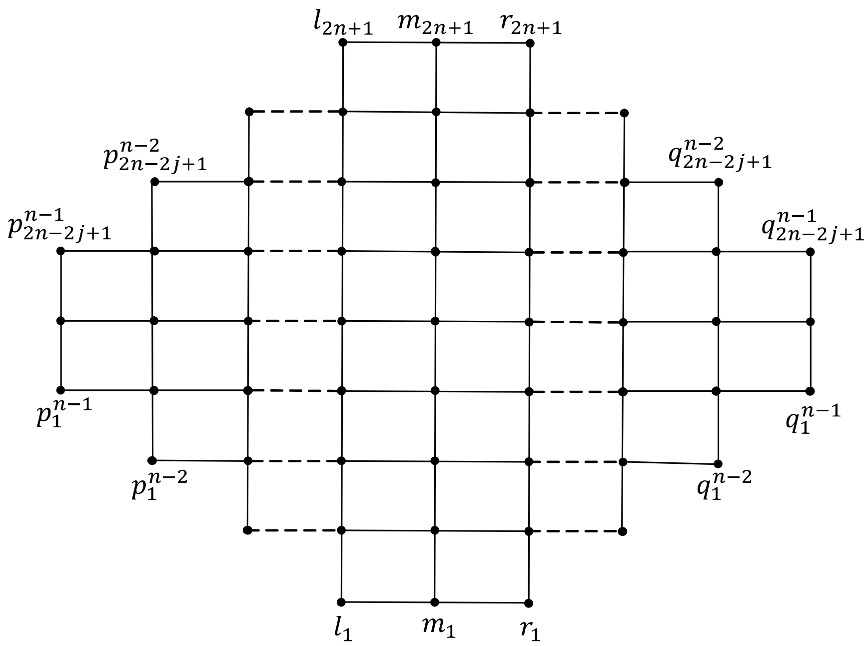

Figure 5: Labeling of Aztec diamond networks of dimension n

By Theorem 4, we have $pd(AZN_n) \geq 3$. We will prove the equality with the following partition-resolving sets. Let $\pi = \{P_1, P_2, P_3\}$, where $P_1 = \{l_1, p_1^1, p_1^2, \ldots, p_1^{n-1}\}$, $P_2 = \{r_1, q_1^1, q_1^2, \ldots q_1^{n-1}\}$ and $P_3 = V(A) - \{P_1 \cup P_2\}$ be the resolving partition set of AZN_n. A vertex's representation in AZN_n about π is as follows.

For $2 \leq i \leq 2n - 2j + 1, 1 \leq j \leq n - 1$ the visual representation of each vertex p_i^j, q_i^j of AZN_n with regard to π as. $r(p_i^j \backslash \pi) = (i - 1, 2j + i + 1, 0)$ and $r(q_i^j \backslash \pi) = (2j + i + 1, i - 1, 0)$.

For $2 \leq i \leq 2n + 1$ each vertex's visual representation r_i, l_i of AZN_n about π as follows.

$r(r_i \backslash \pi) = (i + 1, i - 1, 0)$ and $r(l_i \backslash \pi) = (i - 1, i + 1, 0)$.

For $1 \leq i \leq 2n + 1$ each vertex's visual representation m_i of AZN_n with respect to π as $r(m_i \backslash \pi) = (i, i, 0)$.

According to π the illustration of each vertex in the above graph, we get $r(x \backslash \pi) \neq r(y \backslash \pi)$ for any $x, y \in v(AZN_n)$. Hence $pd(AZN_n) = 3$. □

Theorem 5. *Let A be a Aztec diamond networks of dimension n, then $pd(L(A)) = 3$, $n \geq 2$.*

Proof. Since the vertex set and label of the vertices of $L(AZN_n)$ defined by $V(L(AZN_n) = \{v_0^j; 1 \leq j \leq 2n\} \cup \{v_i^j; i = 1, 2, 3 \ldots, 2n, 1 \leq j \leq 2n + 2\} \cup \{v_{2n+1}^j; 1 \leq j \leq 2n\}$ from bottom to top respectively. The identification of $L(AZN_n)$ is shown in Figure 6.

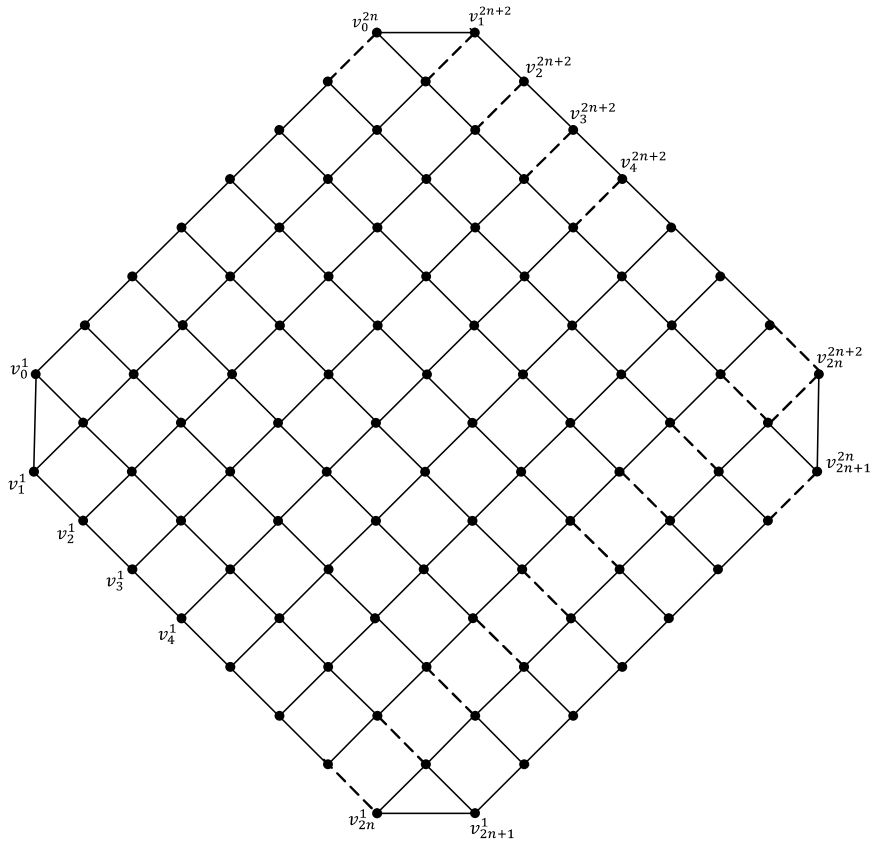

Figure 6: Labeling of the line graph of Aztec diamond networks of dimension n

By Theorem 4, we have $pd(L(AZN_n)) \geq 3$. We will prove the equality with the following partition-resolving sets. Let $\pi = \{P_1, P_2, P_3\}$, where $P_1 = \{v_1^1\}$, $P_2 = \{v_0^j; 1 \leq j \leq 2n\}$ and $P_3 = V(L(AZN_n)) - \{P_1 \cup P_2\}$ be the resolving partition set of $L(AZN_n)$. A vertex's representation in $L(AZN_n)$ about π is as follows.

For $i = 0$, $1 \leq j \leq 2n$ the visual representation of each vertex v_i^j, of $L(AZN_n)$ with regard to π as $r(v_i^j \backslash \pi) = (j, 0, 1)$.

For $i = 1$, $1 \leq j \leq 2n+2$ the visual representation of each vertex v_i^j, of $L(AZN_n)$ with regard to π as $r(v_i^j \backslash \pi) = (j-1, i, 0)$

For $2 \leq i \leq 2n$, $1 \leq j \leq 2n+2$ the visual representation of each vertex v_i^j, of $L(AZN_n)$ with regard to π as $r(v_i^j \backslash \pi) = (i+j-2, i, 0)$

For $i = 2n+1$, $1 \leq j \leq 2n$ the visual representation of each vertex v_i^j, of $L(AZN_n)$ with regard to π as $r(v_i^j \backslash \pi) = (i+j-2, i, 0)$.

According to π the illustration of each vertex in the above graph, we get $r(x\backslash\pi) \neq r(y\backslash\pi)$ for any $x, y \in V(L(AZN_n))$. Hence $pdL(AZN_n) = 3$. □

5 Partition dimension of the extended Aztec diamond networks

The extended Aztec diamond network has the same shape and structure as the original Aztec diamond, with the exception that additional edges join the outside vertices and an n dimensional extended Aztec diamond network denoted by $EAZD_n$. For illustration labeling of an extended Aztec diamond network of dimension n is shown in Figure 7.

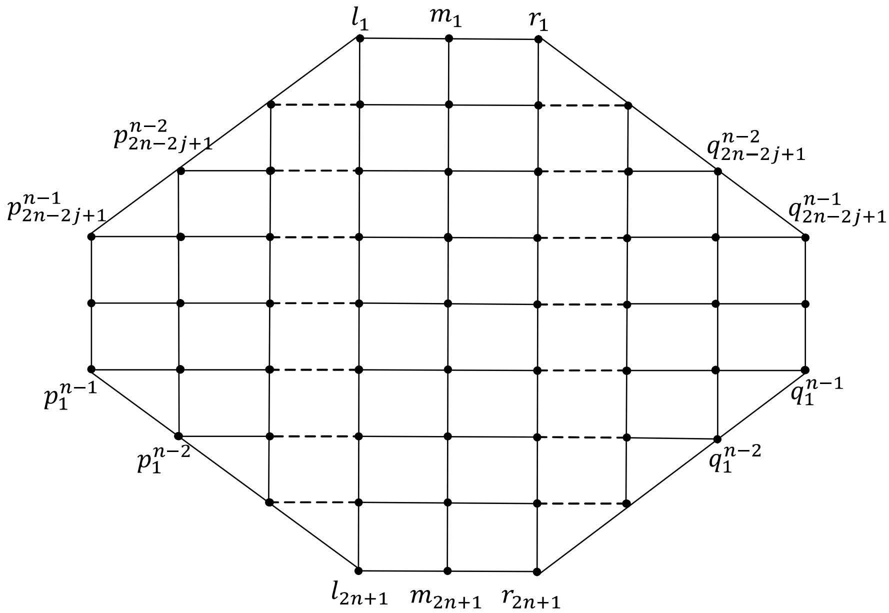

Figure 7: Labeling of extend diamond Aztec networks of dimension n

Theorem 6. *Let A be an extended Aztec diamond network of dimension n, $n \geq 3$ then $pd(EAZD_n) = 3$.*

Proof. Since the set of vertices $EAZD_n$ has 5 levels, say l_i, r_i, p_i^j, q_i^j, and m_i. Label the vertices $\{p_i^j, q_i^j : 1 \leq i \leq 2n-2j+1, 1 \leq j \leq n-1\}$, $\{l_i, r_i : 1 \leq i \leq 2n+1\}$ and $\{m_i : 1 \leq i \leq 2n+1\}$.

By Theorem 4, we have $pd(EAZD_n) \geq 3$. We will prove the equality with the following partition-resolving sets. Let $\pi = \{P_1, P_2, P_3\}$ where $P_1 = \{p_1^{n-1}\}$, $P_2 = \{l_1, m_1, n_1\}$ and $P_3 = V(A) - \{P_1 \cup P_2\}$ be the resolving partition set of $EAZD_n$. A vertex's representation in $EAZD_n$ about π is as follows.

For $2 \leq i \leq 2n+1$ each vertex's visual representation l_i of $EAZD_n$ about π is as follows.

$$r(l_i|\pi) = \begin{cases} (n+1, i-1, 0) & \text{if } 2 \leq i \leq n \\ (n, i-1, 0) & \text{if } i = n+1 \\ (n-1, i-1, 0) & \text{if } n+2 \leq i \leq 2n+1. \end{cases}$$

For $2 \leq i \leq 2n+1$ the illustration of every vertex m_i of $EAZD_n$ about π is as follows.

$$r(m_i|\pi) = \begin{cases} (n+2, i-1, 0) & \text{if } 2 \leq i \leq n \\ (n+1, i-1, 0) & \text{if } i = n+1 \\ (n, i-1, 0) & \text{if } n+2 \leq i \leq 2n+1. \end{cases}$$

For $2 \leq i \leq 2n+1$ the illustration of every vertex r_i of $EAZD_n$ with regard to π as follows.

$$r(r_i|\pi) = \begin{cases} (n+3, i-1, 0) & \text{if } 2 \leq i \leq n \\ (n+2, i-1, 0) & \text{if } i = n+1 \\ (n+1, i-1, 0) & \text{if } n+2 \leq i \leq 2n+1. \end{cases}$$

For $1 \leq i \leq 2n-2j+1, 1 \leq j \leq n-1$ the representation each vertex p_i^j, and q_i^j of $EAZD_n$ with respect to π as follows.

$$r(p_i^j|\pi) = \begin{cases} (n-j+1, i+j-1, 0) & \text{if } 1 \leq i \leq n-1 \\ (n-j, i+j-1, 0) & \text{if } i = n \\ (n-j-1, i+j-1, 0) & \text{if } n+1 \leq i \leq 2n-2j+1, \end{cases}$$

and

$$r(q_i^j|\pi) = \begin{cases} (n+j+3, i+j-1, 0) & \text{if } 1 \leq i \leq n-1 \\ (n+j+2, i+j-1, 0) & \text{if } i = n \\ (n+j+1, i+j-1, 0) & \text{if } n+1 \leq i \leq 2n-2j+1. \end{cases}$$

Each vertex's relationship to π can be seen from the above depiction we get $r(x\backslash\pi) \neq r(y\backslash\pi)$ for any $x, y \in V(EAZD_n)$. Hence $pd(EAZD_n) = 3$. □

6 Final observation

In this article, we have calculated the partition dimension of the line graph of honeycomb network $L(HCN_n)$, Aztec diamond network AZN_n, the line graph of Aztec diamond network AZN_n and extended Aztec diamond network $EAZD_n$ of dimension $n \geq 2$. The resolving partition set can be used as a structure-activity dataset to quickly locate and verify the structure-activity reasons. Moreover, the partition dimension plays a crucial role in understanding the structural and algorithmic properties of line graphs derived from honeycomb networks and Aztec diamond networks. It has applications in communication networks, fault tolerance, tiling problems, algorithmic complexity, and network design, providing valuable insights for various fields ranging from computer science to telecommunications. The obtained resolving partition sets enable easy viewing and browsing of patterns among the used chemical structures through a convenient and intuitive interface. Further partition dimensions of other interconnection networks such as subdivision of Aztec diamond networks and sub division of extended Aztec diamond networks are investigated.

References

Augustine, T., Roy, S. (2023). Radio labeling of certain networks. *Palestine Journal of Mathematics*, *12* (1), 75-80.

Azeem, M., Imran, M. & Nadeem, M.F. (2022). Sharp bounds on partition dimension of hexagonal Möbius ladder. *Journal of King Saud University – Science*, *101779*.

Beerliova, Z., Eberhard, F., Erlebach, T., Hall, A., Hoffmann, M., Mihal'ak, M & Ram, L.S., (2006). Network discovery and verification. *IEEE Journal on Selected Areas in Communications*, *24* (12), 2168–2181.

Chartrand, G., Eroh, L., Johnson, M.A. & Oellermann, O.R. (2000). Resolvability in graphs and the metric dimension of a graph. *Discrete Appllied Mathematics*, *105*, 99-113.

Chartrand, G., Salehi, E. & Zang, P., (1998). On the partition dimension of graph. *Congr. Numer*, *130*, 157–167.

Chvatal, V., (1983). *Mastermind*. Combinatorica, *3*, 325-329.

Fendler, M., Grieser, D. (2015). A simple proof of the Aztec diamond theorem. *Graphs combinatorics*, *32*, 1389–1395.

Harary, F., Melter, R.A., (1976). On the metric dimension of a graph. *Ars Combinatoria*, *2*, 191–195.

Hasmawati, N., Hinding, B., Nurwahyu, A., Daming, S. & Amir, A.K. (2022). The partition dimension of the vertex amalgamation of some cycles. *Heliyon*, *8*.

Hussian, M., Farooq, A. (2019). Metric dimension of line graph of Honeycomb networks. *International Journal of Mathematical and computational Sciences*, *13*, 197-202.

Khuller, S., Raghavachari, B. & Rosenfeld, A., (1996). Landmarks in graphs. *Discrete Appllied. Mathematics*, *70* (3), 217-229.

Koam, A.N.A., Ahmad, A., Azeem, M. & Nadeem, M.F. (2022). Bounds on the partition dimension of one pentagonal carbon nanocone structure. *Arabian Journal of Chemistry*, *15*, 103923.

Kokhas, P. (2009). Domino tilings of aztec diamonds and squares. *Journal of Mathematical Sciences*, *158*, 868-894.

Manuel, P., Rajan, B., Rajasingh, I. & Monica, C. (2008). On Minimum Metric Dimension of Honeycomb Networks. *Journal of Discrete Algorithms*, *6*, 20–27.

Melter, R.A., Tomescu, I. (1984). Metric bases in digital geometry. *Computer vision, graphics, and image Processing*, *25* (1), 113-121.

Monica, M.C., Santhakumar, S., (2019). Partition dimension of rooted product graphs. *Discrete Applied Mathematics*, *262*, 138–147.

Nithya, S., Elavarasi, G. (2022). Partition and local metric dimension of annihilating-ideal graph. *Palestine Journal of Mathematics*, *11*, 98–112.

Rajan, B., William, A., Rajasingh, I., Grigorious, C., & Stephen, S., (2012). On Certain Networks with Partition Dimension Three. *Proceedings of the International Conference on Mathematics in Engineering and Business Management*, 169-172.

Roshini, G.R., Chandrakala, S.B., Vishu Kumar, M. & Sooryanarayana, B. (2021). Non-neighbor topological indices of honeycomb networks. *Palestine Journal of Mathematics*, *10*, 52-58.

Tomescu, I., Javid, I. & Slamin, (2007). On the partition dimension and connected partition dimension of wheels. *Ars Combinatoria*, *84*, 311–318.

Velazquez, J.A.R., Yero, I.G. & Fernau, H. (2014). On the partition dimension of unicyclic graphs. *Bulletin mathématique de la Société des Sciences Mathématiques de Roumanie*, *57*, 381–391.

Velazquez, J.A.R., Yero, I.G. & Lemanska, M. (2014). On the partition dimension of trees. *Discete Appllied Mathematics*, *166*, 204–209

Zhang, X., Bilal, M., Rehman, A., Hussin, M. & Zhang, Z. (2022). On the metric dimension of subdivided honeycomb network and Aztec diamond network. *Discrete Dynamics in Nature and Society*, *2023*, 1-9.